# Random variable algebra in sensor performance modeling

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## Motivation

Decision support tools for signal propagation and sensor modeling typically employ deterministic, physics-based models of signal propagation, but are frequently forced to use **imperfect** or **uncertain** input data for analysis. Here we show that these uncertainties can have significant impacts on **sensor performance estimation** and modeling **environmental effects on propagated signals**.

## Simple Sensor Model

We have constructed a sensor system model in an effort to understand the impact of simple algebraic relationships between uncertain inputs. For simplicity, these inputs have been chosen as either uniform (U) or lognormal (L) distributions.

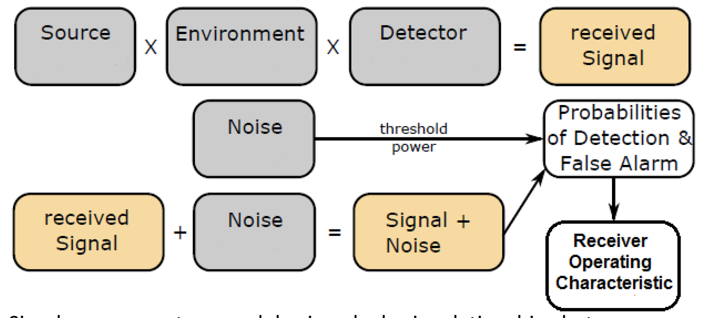


**Figure 1:** Simple sensor system model using algebraic relationships between components. Grey components show uncertain (statistically represented) inputs to the system model, while orange components are quantities derived from the inputs, which must also be uncertain, and yet are deterministically related to the inputs. The deterministic relationships are governed by **random variable algebra**.

## Random variable algebra

Received Signal and Signal + Noise distributions are calculated for 4 different cases using numerical, random variable algebra techniques.

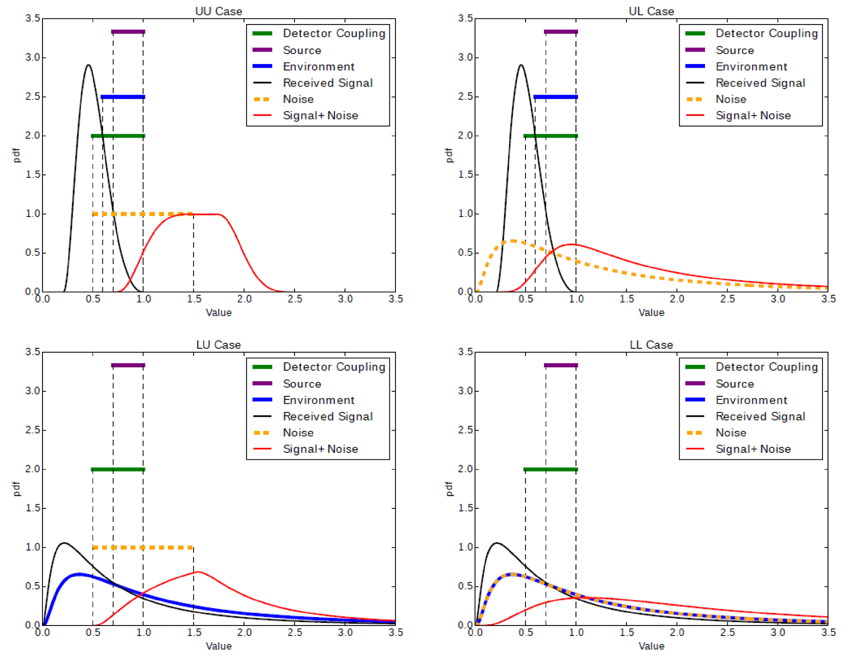

**Figure 2:** Input and calculated probability density functions (pdfs) for four different cases:
UU = environment and noise distributions are uniform
UL = environment is uniform, noise is lognormal
LU = environment is lognormal, noise is uniform
LL = environment and noise distributions are lognormal.
Signal and Detector Coupling distributions are uniform in all cases..

## Sensor Performance Modeling

Using complementary cumulative distribution functions derived from the statistical sensor model, we can determine probabilities of detection ($P_d$) and false alarm ($P_{fa}$) as a function of signal and noise powers for our four cases.

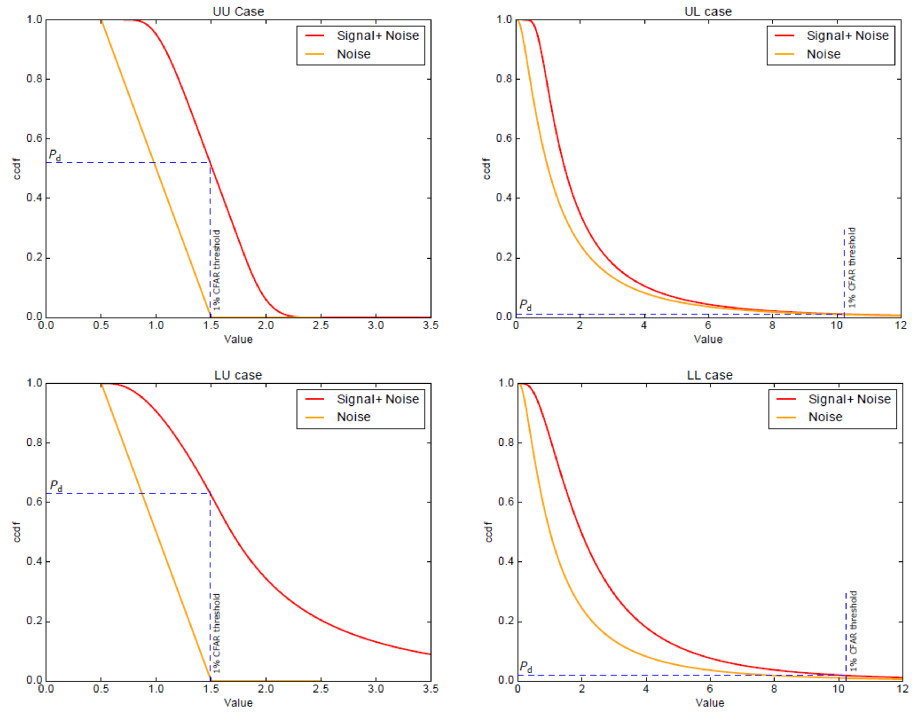

**Figure 3:** Determining probabilities of detection ($P_d$) and false alarm ($P_{fa}$) for the simple sensor sytem in each of the four (UU, UL, LU and LL) cases. Note the difference in x-axis range due to the heavy tails of cases involving lognormal distributions.

The resulting ($P_d$ ,$P_{fa}$) pairs are used to build receiver operating characteristic (ROC) curves for each case. **Better performing sensors** have ROC curves closer to the upper left corner, where $P_d$ is high and $P_{fa}$ is low.

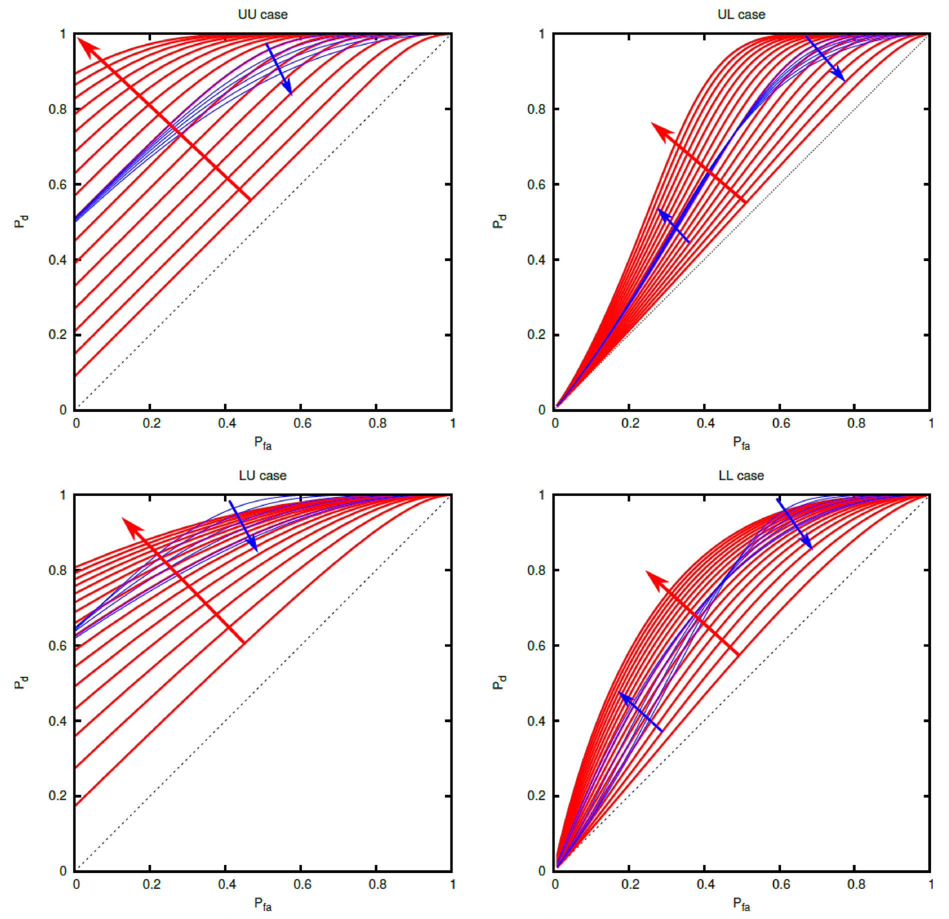

**Figure 4:** Sensor performance ROC curves for four different cases. A single ROC curve corresponds to a single set of input distributions. Additional curves in each case show the impact of increasing Signal power (red) and increasing Environment variance (blue), simulating the case where the general shape of an input distribution is known, but uncertainty exists in its moments (mean, variance, etc).
Red curves progress in direction of increasing Signal power, indicated by red arrow.
Blue curves progress in direction of increasing Environment variance, as indicated by blue arrow.

## Computational Costs

Two different numerical techniques were used to perform random variable algebraic operations:

- PaCAL (Korzen & Jaroszewicz, 2012), implemented in Python, uses **Chebychev polynomials** for pdfs
- `rvlib` (Lourens & van Geer, 2014), implemented in an R library in Fortran, uses **linear functions** to represent pdfs

Both methods use piecewise-continuous functions to represent the pdfs, but with substantial differences in performance.

| Case | Monte Carlo | PaCAL | rvlib |
|---|---|---|---|
| UU | 724 | 1.61 | 0.22 |
| LU | 724 | 3.85 | 0.40 |
| UL | 724 | 5.54 | 0.25 |
| LL | 724 | 8.67 | 0.41 |

**Figure 5:** Single-core CPU computational times, in seconds, to determine a 100 point ROC curve.

Does the computational speed advantage of using `rvlib` cost us accuracy? Comparing PaCAL and `rvlib` against a "gold standard" Monte Carlo result, we found that for the operations and input distributions used in this work, **the calculation accuracy was the same**.

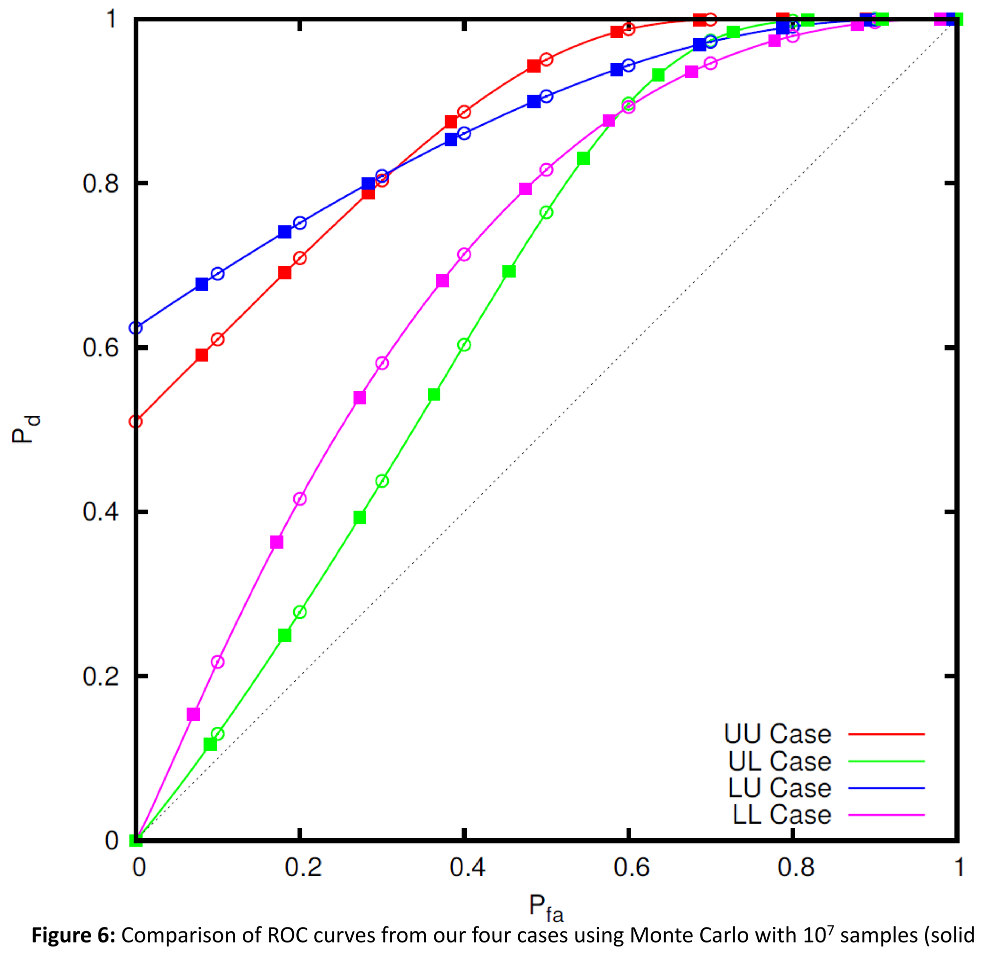


**Figure 6:** Comparison of ROC curves from our four cases using Monte Carlo with $10^7$ samples (solid lines), PaCAL (solid squares), and rvlib (open circles). Only a few points are shown, for clarity, but agreement was excellent, usually better than 0.05% between the three different techniques.

## Measured environmental effects on propagated radiofrequency signals

To test the Received Signal portion of the simple sensor system model, we conducted a non-line-of-sight radiowave propagation experiment using

- fixed power source (delta function), to statistically characterize the channel propagation losses, and
- "uncertain" source power, statistically defined (double triangular distribution)

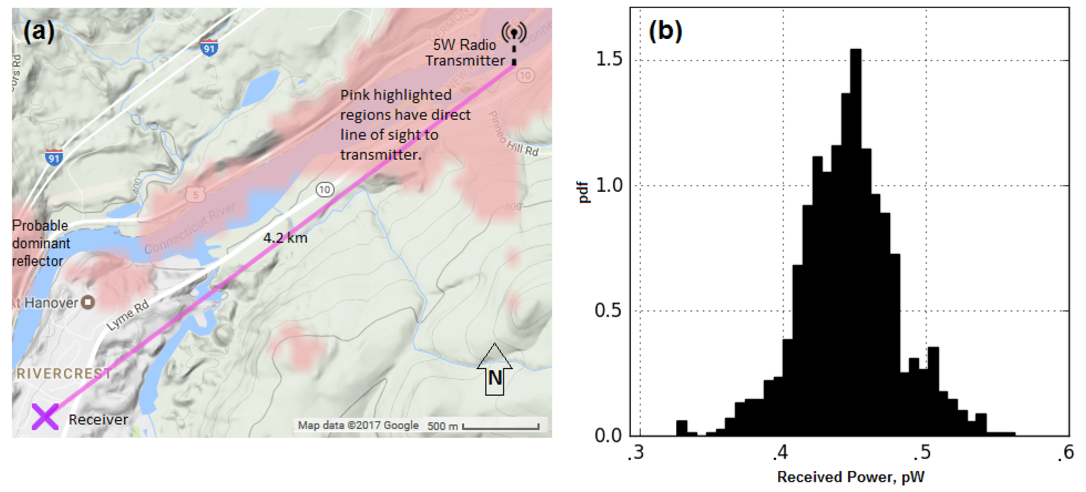


**Figure 7:** (a) Map of propagation experiment, (b) received power for fixed power source, as determined by N=1000 separate signal transmissions. The distribution of received power captures the Environment used to derive the Received Signal power distribution.

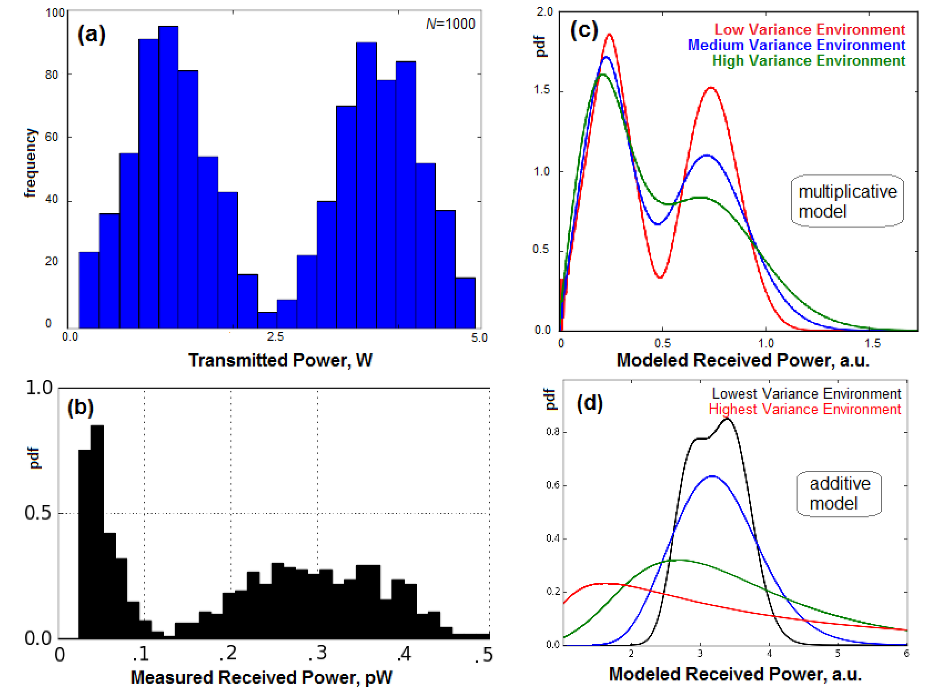


**Figure 8:** (a) Distribution of "uncertain" Source power actually transmitted. (b) Measured Received power distribution after propagation of N=1000 separate signal transmissions through the Environment shown in Figure 7. The lower power peak is partially truncated by the receiver noise floor. (c) Prediction for Received power determined by random variable algebra multiplicative model. (d) Prediction for additive model.

The multiplicative model (i.e. the one used in Figure 1 and shown in 8(c) ) for Received Signal power predict a narrow, peak in the pdf for low powers, and a lower, wider peak for high powers, which is consistent with our measured data. The additive model is clearly wrong, failing to even capture the multi-modes in the real data.

## Conclusions

Sensor system models using random variable algebra can be used to explicitly include uncertainties in inputs, and significantly impact sensor performance estimates. Preliminary testing shows random variable algebra techniques are applicable to modeling signal propagation problems with realistic levels of uncertainty.

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